Muon Collider Parameters

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Muon Collider Physics Workshop

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- Parameters
- Luminosity
- Collider Ring
- Cooling → emittances & transmission
- Production → Proton Energy
- Beam loading and wakes in muon acceleration
- Neutrino radiation → depth & beam energy constraint
- Conclusion

Parameters

C of m Energy	1.5	3	TeV
Luminosity	0.92	3.4	10^{34} cm^{2}sec^{-1}
Beam-beam Tune Shift	≈0.087	≈0.087	
Muons/bunch	2 (1.44 ?)	2	10^{12}
Total muon Power	9	15	MW
Ring $<$bending field$>$	6	8.4	T
Ring circumference	2.6	4.5	km
β^* at IP $= \sigma_z$	10	5	mm
rms momentum spread	0.1 (0.3 ?)	0.1	%
Required depth for ν rad	≈20	≈ 200	m
Proton Energy	8	8	GeV
Muon per proton	0.16	0.16	
Muon Survival	7	6	%
protons/pulse	187 (134 ?)	200	Tp
Repetition Rate	15 (21 ?)	12	Hz
Proton Driver power	≈3.5	≈3	MW
Muon Trans Emittance	25 (18 ?)	25	pi mm mrad
Muon Long Emittance	72k (210k ?)	72,000	pi mm mrad

The 3 TeV numbers are far less studied than the 1.5 TeV ones

The numbers keep changing & remain uncertain We just do not know enough

Luminosity Dependency

$$\mathcal{L} \;=\; n_{\rm turns}\; f_{\rm bunch}\, \frac{N_\mu^2\;\gamma}{4\pi\epsilon_\perp\;\beta^*}\;F_2 \qquad\qquad \Delta\nu \;=\; \frac{N_\mu\;r_o}{4\pi\epsilon_\perp}\;F_1$$

F_1 and F_2 depend on the hourglass effect and 'Disruption' enhancement. Both are of order 1.0, so approximately:

$$\mathcal{L} \;\propto\; B_{\rm ring}\; P_{\rm beam}\;\; \Delta\nu\; \frac{1}{\beta^*}$$

- Beam beam tune shift $\Delta\nu$, if too large, causes beam loss
- Simulations and electron experience suggests a limit of

$$\Delta\nu \;\le\; 0.1$$

- If this can be achieved then
  - Luminosity does NOT depend on emittance
  - Luminosity $\propto$ Beam Power, bending fields, and small β^*

Collider ring design

- It must accept the transverse emittance with 3-5 sigma
- It should have the smallest possible β^* (tight focusing at IP)

$$\mathcal{L} \propto \frac{1}{\beta^*}$$

- It should have the smallest circumference $\longrightarrow$ the highest average bending field

$$\mathcal{L} \propto \frac{1}{\mathrm{circ}} \ \propto \ < \mathrm{B}_{\mathrm{bending}} >$$

- It should have the largest possible momentum acceptance

$$\epsilon_{\parallel} \ = \ \frac{\sigma_z \ dp/p}{\beta\gamma} \approx \frac{\beta^* \ dp/p}{\beta\gamma}$$

The larger $\epsilon_{\parallel}$ the smaller we can make $\epsilon_{\perp}$ for the same ϵ_6

The latest lattice (Eliana's) has dp/p acceptance $\longrightarrow$ 1 % which would allow $\sigma_p/p: \quad 0.1 \longrightarrow$ 0.3 % and might allow lower $\epsilon_{\perp}$ (see later)

Dependencies on Transverse Emittance

Luminosity may not depend on emittance BUT

$$N_\mu = \frac{4\pi \ \Delta\nu \ \epsilon_\perp}{r_o} \ \propto \ \epsilon_\perp$$

$$N_p = \frac{N_\mu}{\eta_{\mu/p} \ \eta_{\rm survival}} \ \propto \ \epsilon_\perp$$

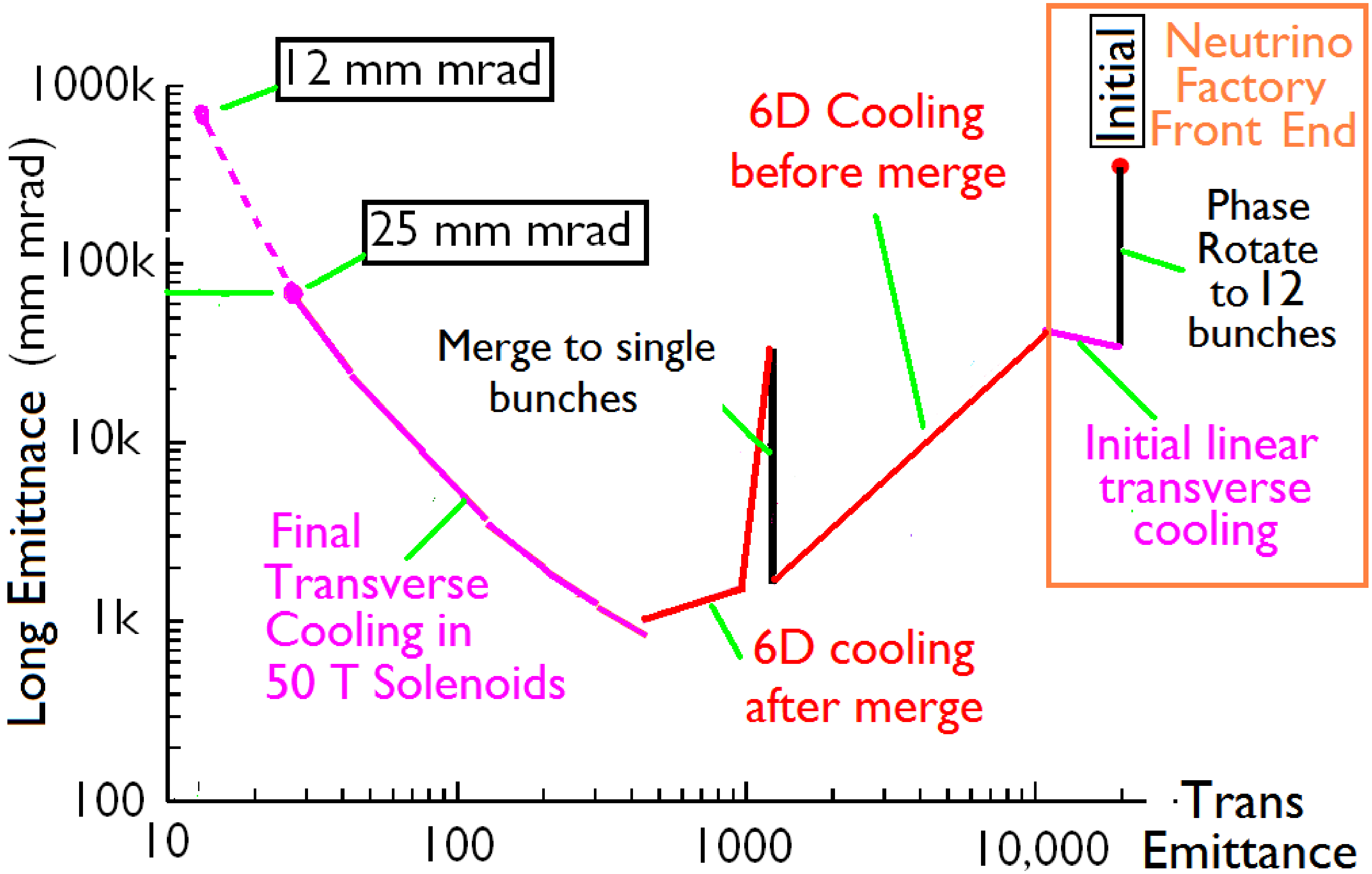
$$f_{\rm rep} = \frac{P_{\rm beam}}{E_\mu \ N_\mu} \ \propto \ 1/\epsilon_\perp$$

$\epsilon_\perp$	12	25	50	π mm mrad
N_μ	1	2	4	10^{12}
N_p	93	187	374	Tp
$f_{\rm rep}$	30	15	7	Hz

For 1.5 TeV (c of m)

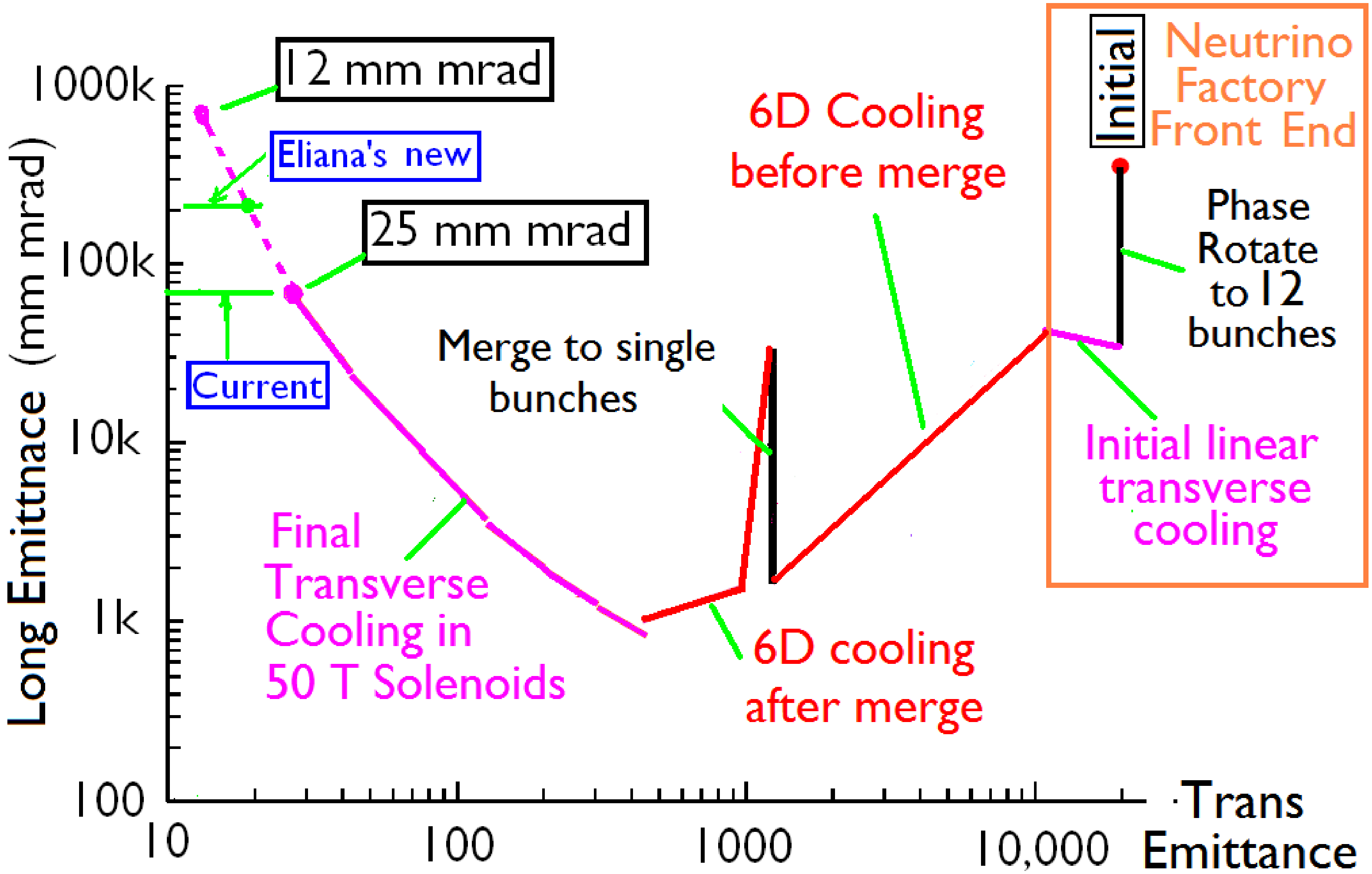
- $N_\mu = 4 \ 10^{12}$ would require lower rf frequency to accelerate
- $N_p = 374$ Tp $\rightarrow$ Space charge problems in buncher
- But getting $\epsilon_\perp \ < \ 25$ is hard

Cooling → Emittances and transmission



- Every stage simulated at some level, But with many caveats
- Final $\epsilon_\perp = 25$ (mm mrad) obtained in 50 T HTS solenoid
- 12 (mm mrad) gives 10 $\times$ Required Longitudinal emittance

Parameters using Eliana's New Lattice

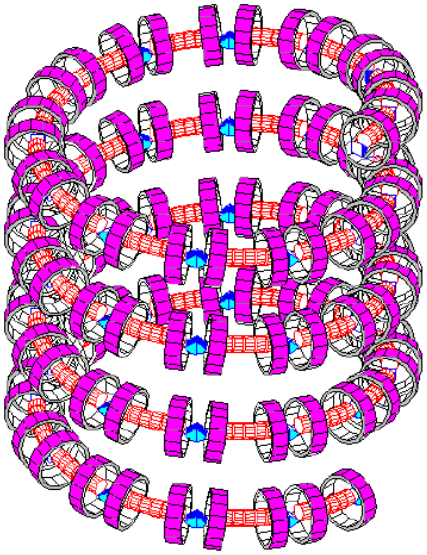


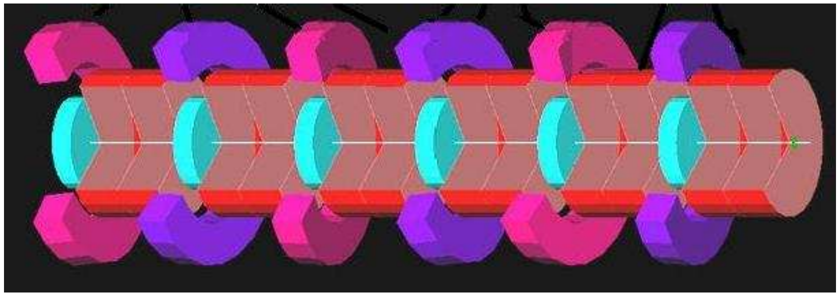
$\epsilon_\perp$	25	18	π mm mrad
N_μ	2	1.44	10^{12}
N_p	187	134	Tp
$f_{\rm rep}$	15	21	Hz

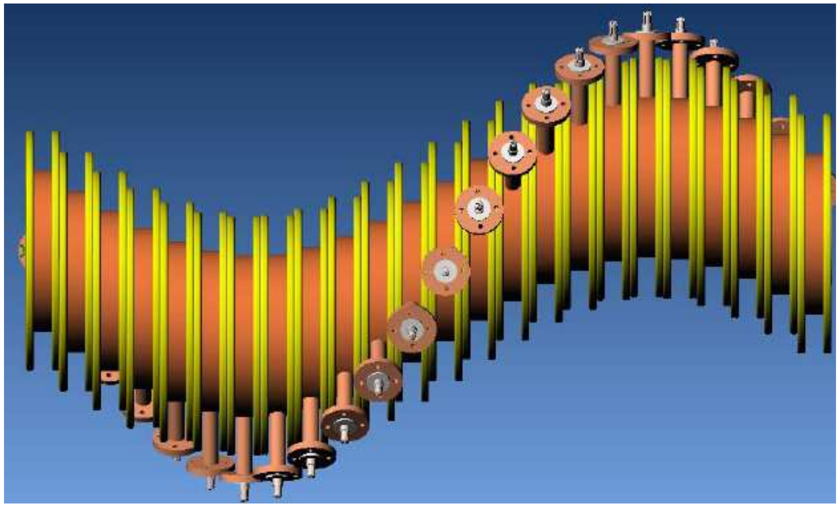
- A not negligible gain
- But involves possible difficulties in final cooling

6D cooling Methods

- Several proposed schemes:
  - RFOFO Guggenheim
  - FOFO Snake
  - HCC
- They all have rf, focusing, absorbers
- They all work in simulation
- They could be used interchangeably
- The choice will depend on technical questions no yet resolved







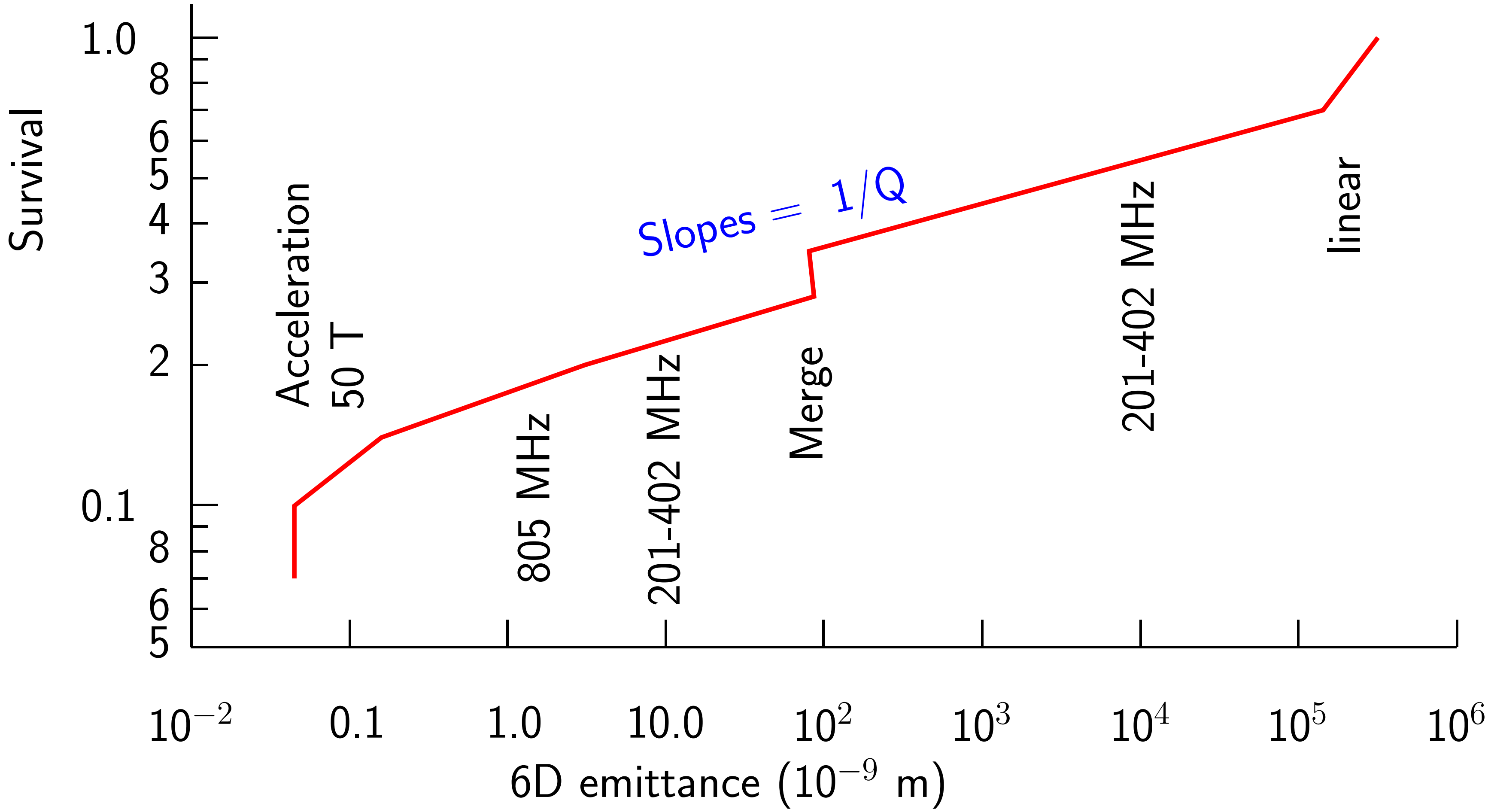
- An important parameter is

$$Q = \frac{d\epsilon_6/\epsilon_6}{dN/n}$$

because $$\frac{N_{\rm final}}{N_{\rm initial}} = \left(\frac{\epsilon_6 \quad {\rm initial}}{\epsilon_6 \quad {\rm final}}\right)^{1/Q}$$

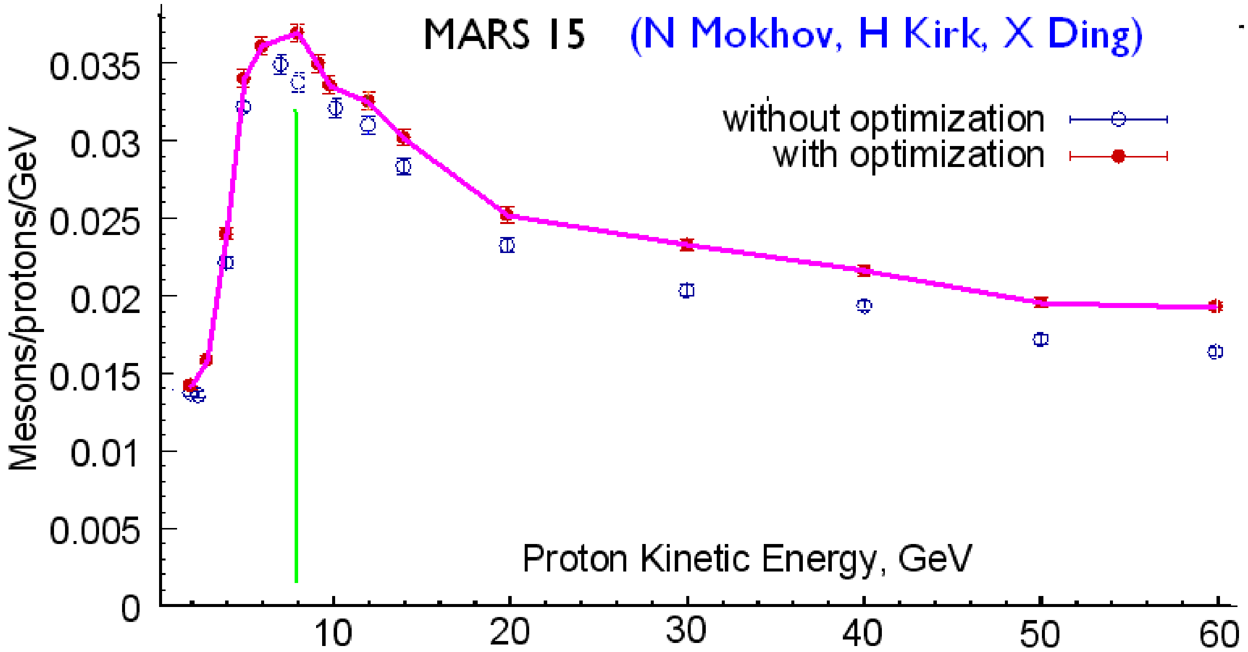
Estimated transmission → p intensities & power

Based on simulations plus relatively optimistic estimates of matching

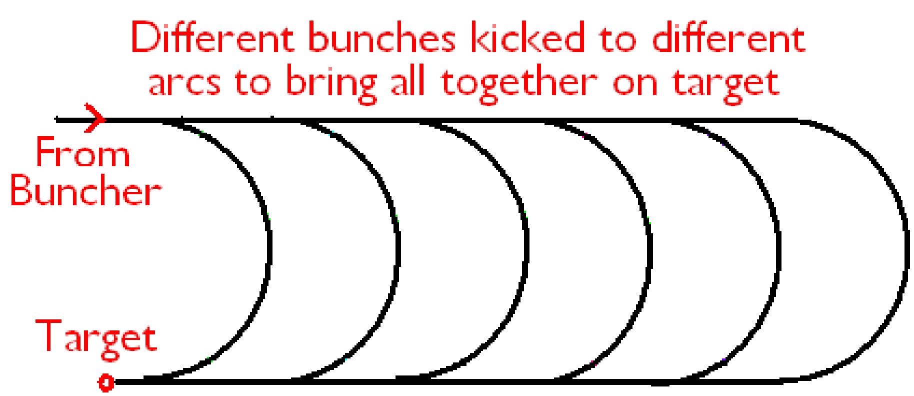


- Only ≈ 7% survive (only part of this loss is from decay)
- This means that the initial pion, and thus proton, bunches must be intense

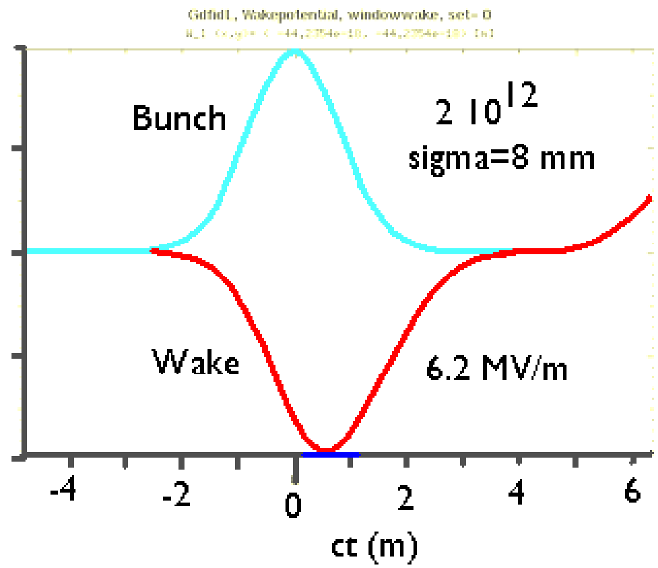
Meson Production → Proton Energy



- Maximum production at 8 GeV
- Requires ≈ 200 Tp in 2 nsec → Severe space charge tune shift in buncher
- But appears possible with 6-8 bunches & trombones (Ankenbrandt)

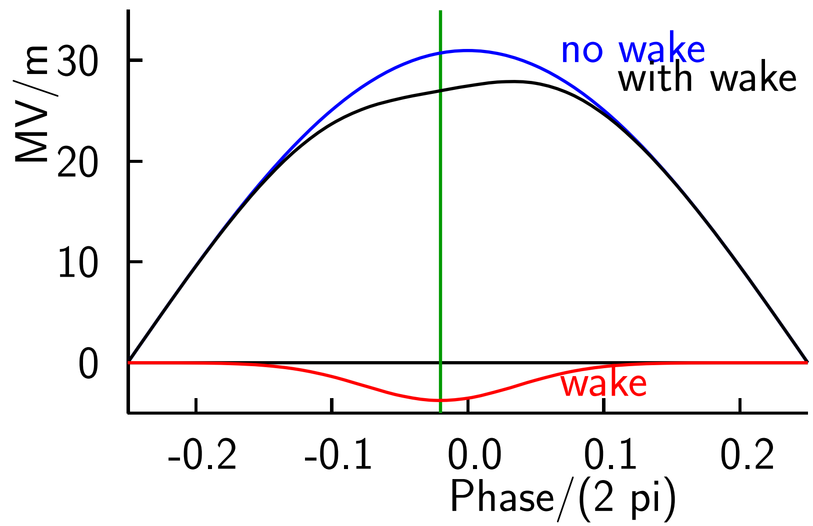


Loading of Muon Acceleration



- Short range longitudinal wake for 1.3 GHz ILC cavities & σ_z=13.2 mm rms
- Wake=3.75 MV (Yakovlev)
- If 31 MV/m = 12% wake

- Is corrected by rf waveform (Balakin)

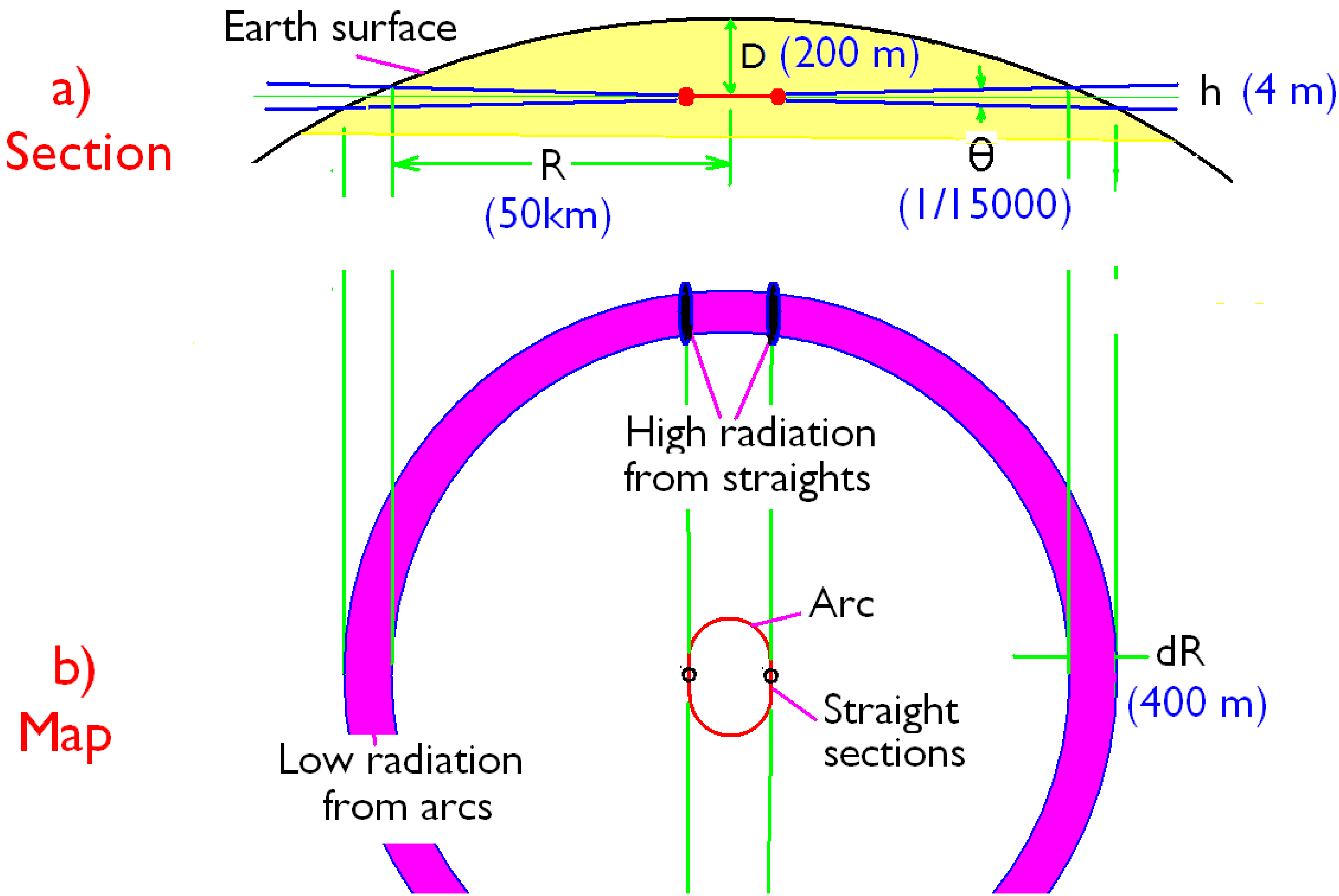


Beam Loading

- Uses ≈ 10 % of rf energy for each transit of both charges
- Giving good efficiency but requiring high rf power to maintain gradient
- 2 10^{12} muons per bunch does not look impossible

Neutrino Radiation

Numbers in paretneses for 3 TeV



Neutrino Radiation Constraint on Beam Power

$$\text{Radiation} \ \propto \ \frac{E_\mu \ I_\mu \ \sigma_\nu}{\theta \ R^2} \ \propto \ \frac{P_{\text{beam}} \ \gamma^2}{D}$$

- For $\mathcal{L} \propto \gamma^2$ then: Radiation $\propto \gamma^4$
- Little problem at 1.5 TeV
- Depth of order 200 m for 3 TeV ring

- But muon power must be constrained
- straights must be avoided $\rightarrow$ combined function magnets
- Probably practical to "own" areas of radiation from IP straight sections

Parameters again

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The 6 TeV numbers are a blind extrapolation with the same ν radiation
One should investigate going deeper, $\Delta\nu$ correction, higher $< B >$

Conclusion

- All parameters are tightly inter-connected
- They keep changing as we learn more, and remain uncertain
- The least certain parameter is probably the muon survival in cooling giving uncertainties in:
  - proton bunch intensity
  - required repetition rate
  - luminosity
- We also do not yet have a design for the 3 TeV ring so we do not know
  - β^* and thus luminosity
  - dp/p and thus longitudinal emittance requirement
- Much work to be done